Wave-Particle duality at the Planck scale: Freezing of neutrino oscillations

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Abstract

A gravitationally-induced modification to de Broglie wave-particle duality is presented. At Planck scale, the gravitationally-modified matter wavelength saturates to a few times the Planck length in a momentum independent manner. In certain frameworks, this circumstance freezes neutrino oscillations in the Planck realm. This effect is apart, and beyond, the gravitational red-shift. A conclusion is drawn that in a complete theory of quantum gravity the notions of "quantum" and "gravity" shall carry new meanings – meanings, that are yet to be deciphered from theory and observations in their entirety.

1 Introduction

One of the most challenging quests in contemporary theoretical physics concerns the nature of space-time at the Planck scale, and deciphering gravitationally-induced modifications to the quantum realm, and vice versa. While some of these aspects can only be revealed by observations, Maxwellian arguments of consistency can also shed light on the joint realm of the gravity and the quantum.

One such Maxwellian argument was presented in Ref. [1]. It says that quantum measurements in the Planck realm necessarily alter the local space-time metric in a manner that destroys the commutativity of the position measurements of two different particles. In addition, it also affects the fundamental commutator,

$$[x, p_x] = i\hbar \tag{1}$$

The essential idea of the above argument resides in the observations that a position measurement collapses the wave function, say, in the following manner, Position Measurement : $\langle \vec{r} | \psi (0 \leq r \leq \infty) \rangle \rightarrow \langle \vec{r} | \psi (0 \leq r \leq R) \rangle$. In case R is of the order of Planck length, the gravitational effects associated with the wave function collapse become important as it necessarily invokes the collapse of the energy-momentum tensor. Hence, the local space-time metric changes. As shown in Ref. [1], this circumstance makes the position measurements of two distinguishable particles non-commutative.

As a consequence, non-locality must be an essential part of any attempt to merge the theory of general relativity with quantum mechanics. The derived non-commutativity easily extends to measurements of different components of the position vector of a single particle, and modifies the fundamental commutators of the Heisenberg algebra. A further essential conclusion beyond the stated gravitationally-induced non-locality is that space-time itself acquires a non-commutative character.

Some implications of such non-commutative space times have been studied, e.g., by Madore [2], and by Connes [3], however, from an entirely different view point. Independently, efforts in string theories also arrive at gravitationally-modified fundamental uncertainty relations. In that context an early reference is the work of Veneziano [4], while a recent one is [5]. In yet another line of argument, without invoking extended objects, and entirely within the framework of quantum mechanics and the theory of general relativity, Adler and Santiago [6] also obtain similar modifications to the uncertainty principle without invoking extended objects (*cf.*, [7–10]). A somewhat different argument, based on the existence of an upper bound for acceleration, also results in a gravitational modification to the uncertainty principle [11]. The mathematical expression of the above results that leads to a gravitationally modified expression for the wave particle duality is given by the following modification to the fundamental commutator [12]:

$$[\mathbf{x}, \, \mathbf{p}] = i\hbar \left[\mathbf{1} + \epsilon \frac{\lambda_P^2 \mathbf{p}^2}{\hbar^2} \right] \tag{2}$$

where $\lambda_P = \sqrt{\hbar G / c^3}$, is the Planck length, and ϵ is some dimensionless number of the order of unity. In what follows I set ϵ equal to unity.

It is the purpose of this *Letter* to decipher the wave-particle duality as contained in (2). To make our argument, we first recapture the origin of the wave-particle duality in the absence of gravitational effects, and then immediately return to the stated objective.

1.1   Wave-Particle duality in the absence of gravity

The fundamental commutator, (1), encodes the fact that intensity of matter and gauge fields cannot be arbitrarily reduced to zero, but is bounded from below. The first direct evidence for this circumstance came from Einstein's understanding of the photo-electric effect. It is precisely this commutator that lies behind the de Broglie relation, and the entire edifice of the wave-particle duality. To see this, recall that in configuration space, $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$, is a solution of the fundamental commutator (1), with eigenfunctions of the form $\psi(p_x) = N \exp\left(\frac{i}{\hbar}p_x x\right)$. The spatial periodicity, $\lambda = \frac{h}{|p_x|}$, carried by $\psi(p_x)$, when extended to three dimensions, yields the well known de Broglie relation

$$\lambda = \frac{h}{p},\tag{3}$$

where $p = |\vec{p}|$ is the magnitude of the momentum vector associated with an object. A simple text-book algebraic exercise, with (1) as the physical input, gives the Heisenberg uncertainty relation

$$\Delta x \Delta p_x \geq \hbar/2, \text{ etc.}\tag{4}$$

In the absence of gravity, equations (1), (3), and (4) represent various inter-related aspects of the wave-particle duality. One immediately sees that as p approaches the Planck scale, and then beyond, the de Broglie wavelength continuously shrinks to zero and allows quantum-mechanical probing of space-time to all length scales and energies. However, as already mentioned, if the gravitational effects in the quantum-measurement process are taken into consideration, these results are no longer true. Planck length, up to a factor of the order of unity, emerges as the limiting length scale beyond which space-time cannot be probed. This circumstance, therefore, immediately suggests that the the relation (3) must undergo a change in which the left hand side of (3) saturates to, within a few times, the Planck length. It is precisely this that emerges in the following.

As long as the entire theoretical structure of the existing quantum field theories rests upon the wave-particle duality, it is necessary to fix the domain of its validity. The heaviest objects for which the wave-particle duality (3) has been experimentally verified, so far, is the C_{60} fullerene [13]. In this context, the experiment sets the scale at $m_{C_{60}} = 1.20 \times 10^{-21}$ g. This is already an impressive achievement. Yet, it is to be compared with the Planck mass, $m_{Pl.} \equiv (\hbar c/G)^{1/2} = 2.18 \times 10^{-5}$ g. However, in order to study possible departures from the de Broglie wave-particle duality in the Planck regime, one may even not need to invoke early universe directly. All one may need are Planck

mass quantum objects, and an appropriate technique to study an associated interference phenomena. To be specific, such effects may become indirectly observable via extremely high-energy gamma rays, and high-energy neutrinos.

Here, and in the following, the operator, or c-number, nature of objects, such as p_x in (1), where it is an operator, and p_x in $\psi(p_x)$, where it is a c-number, shall be omitted and will be assumed apparent from the context.

2 Gravitationally-modified Wave-Particle duality: minimal modification, and some implications

In one spatial dimension (chosen as x), the gravitationally-modified position-momentum uncertainty relation immediately follows from the commutator (2), and reads:

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2} \left[1 + \left(\frac{\lambda_P \Delta p_x}{\hbar} \right)^2 + \left(\frac{\lambda_P \langle \mathbf{p} \rangle}{\hbar} \right)^2 \right]. \tag{5}$$

It carries as a characteristic feature the Kempf-Mangano-Mann (KMM, ref.[12]) lower bound on the position uncertainty:

$$\Delta x_{K} = \lambda_P \left(1 + \frac{\lambda_P \langle \mathbf{p} \rangle}{\hbar} \right)^{1/2} \tag{6}$$

Notice that Δx_K has a state dependence via $\langle \mathbf{p} \rangle$. For a state of a vanishing $\langle \mathbf{p} \rangle$, one obtains the absolute minimal distance that can be probed quantum mechanically. This lowest bound does not depend on the particle species. Therefore, the existence of the "absolute minimal distance" suggests a new intrinsic property of the space-time itself.

An important implication of the KMM lower bound, Δx_{K}, is that the de Broglie plane waves can no longer represent the physical wave functions, even not in principle. Thus the wave-particle duality must undergo a fundamental conceptual and quantitative change.

A non-relativistic modification to the de Broglie relation was presented in pioneering KMM work. This case, however, is likely to be of limited interest in the Planck regime. Here, I present the gravitationally modified de Broglie relation without restrictions on the particle's momentum.

It is readily seen that the momentum space wave function consistent with the gravitationally modified uncertainty relations (5) reads [12]:

$$\psi(p) = N \left(1 + \beta p^2\right)^{-\left[\kappa(\mathbf{p})/4\beta(\Delta p)^2\right]}$$
$$\times \exp\left[-i\frac{\langle \mathbf{x} \rangle}{\lambda_P}\tan^{-1}\left(\sqrt{\beta}p\right) - \frac{\kappa(\mathbf{p})\langle \mathbf{p} \rangle}{2(\Delta p)^2\sqrt{\beta}}\tan^{-1}\left(\sqrt{\beta}p\right)\right] \qquad (7)$$

where $\kappa(\mathbf{p}) := 1 + \beta(\Delta p)^2 + \beta\langle \mathbf{p} \rangle^2$, and $\beta := \lambda_P^2/\hbar^2$. N is a normalization factor. This represents an oscillatory function damped by a momentum-dependent exponential. I identify the oscillation length with the gravitationally modified de Broglie wave length:

$$\lambda = 2\pi \frac{\lambda_P}{\tan^{-1}\left(\sqrt{\beta}p\right)} \qquad (8)$$

Introducing $\overline{\lambda}_P := 2\pi\lambda_P$ as the *Planck circumference*; and λ_{dB} as the gravitationally **un**modified de Broglie wave length, $\lambda_{dB} = h/p$, the above expression takes the form:

$$\lambda = \frac{\overline{\lambda}_P}{\tan^{-1}\left(\overline{\lambda}_P/\lambda_{dB}\right)} \begin{cases} \rightarrow \lambda_{dB} & \text{for low energy regime} \\ \rightarrow 4\lambda_P & \text{for Planck regime} \end{cases} \qquad (9)$$

In addition, for the specific non-relativistic regime considered by Kempf *et al.* [12], λ reproduces their equation (44). This justifies the interpretation of the oscillatory length associated with KMM's $\psi(p)$ as the gravitationally modified de Broglie wavelength.

The gravitationally induced modifications to (1), (3), and (4) are now contained in (2), (9), and (5). These latter equations constitute the minimal conceptual and quantitative changes in the nature of the wave-particle duality.

A brief discussion on immediate physical implications of the modified wave-particle duality in the Planck realm now follows.

2.1 Freezing of neutrino oscillations

To explore one of the concrete consequences of the above-presented modification to the wave-particle duality, note that the existing data suggests flavor eigenstates of neutrinos to be linear superposition of different mass eigenstates [14]:

$$|\nu_\ell\rangle = \sum_{j} U_{\ell j}|m_j\rangle \tag{10}$$

where, $\ell = e, \mu, \tau$, is the flavor index, and $j = 1, 2, 3$, enumerates the mass eigenstates, while U is a 3×3 unitary matrix. Several fundamental questions now arise. Is this low-energy, i.e. low in comparison to the Planck mass, construct still valid at the Planck scale? What is the time evolution of the flavor and mass eigenstates in the Planck realm? At a deeper and related level, does the non-commutative space-time still carry Poincaré symmetry? – for the very notions of mass and spin (which the underlying mass eigenstates carry) originate from the Casimir invariants associated with the Poincaré group. In addition, the equations governing the evolution of the states derive their form from the space-time symmetries. None of these questions has a readily available answer. An answer must, therefore, await future theoretical and observational input. The latter, for example, may come from the study of anomalous events around and beyond 10^{20} eV cosmic rays.

Under these circumstances we take note of the fact that low-energy neutrino oscillations owe their physical origin to different de Broglie oscillation lengths associated with each of the underlying mass eigenstates. If one assumes that each of the mass eigenstates carries the same energy, then the flavor oscillations arise due to different de Broglie oscillation lengths carried by each of the mass eigenstates. If this scenario was considered for neutrino oscillations then it is clear that neutrino oscillations shall freeze at Planck scale due to the above obtained gravitationally-induced modification to the wave-particle duality. In particular, I draw attention to the saturation of λ as indicated in Eq. (9).

In the ordinary neutrino oscillation phenomenology the flavor oscillations are not altered at any practical level if one considers the "equal energy," or "equal velocity", or "wave packet" approaches [15–18]. Not knowing the answer to the questions posed above it is not yet possible to say if the Planck-scale freezing of neutrino flavor oscillations shall survive in all neutrino oscillation frameworks.

Our discussion here, therefore, is intended to bring attention to the fact that the gravitationally-induced modifications to the wave-particle duality may have significant physically observable consequences for the early universe.

2.2 Effect on H-atom

For comparison, to the lowest order in λ_P, the effect of the modification (5) on the ground state level of the hydrogen atom results in the following modified uncertainty principle estimate for the ground state of an electron in an H-atom:

$$(E_0)_g \simeq -\frac{me^4}{2\hbar^2}\left[1 - \frac{4m\lambda_P^2}{\hbar^2}\left(\frac{me^4}{2\hbar^2}\right)\right] \qquad (11)$$

Identifying:

$$E_0 = -\frac{me^4}{2\hbar^2} \qquad (12)$$

with the ground state level of the hydrogen atom without incorporating the gravitationally-induced correction to the uncertainty relation, one immediately notices that the effect of gravitational corrections is to reduce the magnitude of the ionization energy by 2.5×10^{-48} eV. This suggests that a space-time endowed with the KMM bound is in some sense a heat bath as it decreases the energy required to disassociate the H-atom.

2.3 Coherence in the early universe and in biological systems

The wavelength λ asymptotically approaches the constant value $4\lambda_P$ that is now universal for all particle species. As a consequence of this universality, a new type of coherence may emerge in the early universe and this may carry significance for the large-scale uniformity of the universe. It is also speculated that quantum mechanics plays a fundamental role in brain function, see,e.g., [19]. Therefore, the new coherence may also carry significant implications for functioning of the brain, and other biological systems, if important biological elements carry a mass of the order of $M_P = (\hbar c/G)^{1/2} = 2.2 \times 10^{-5}$ g.

3 Conclusion

If the effects of the gravitationally-induced modification to the de Broglie wave particle duality are negligible for low energy, their relevance perhaps cannot be overestimated at the Planck-scale. At present, there are already speculations that anomalous events around 10^{20} eV cosmic rays may be pointing towards a violation of the Lorentz symmetry [20,21]. It is expected that the gravitationally-modified wave-particle duality carries with it deformations of the Poincaré symmetries. Some of these deformations can be studied with the recently-approved Gamma-ray Large Area Space Telescope (GLAST), and with other detectors. [1]

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[1] The reader is referred to references [22–24] for the original proposal, and for details on the recent progress in this direction. A related proposal on gravitationally-induced modification of quantum evolution by Ellis, Hagelin, Nanopoulos, and Sred-

In the context of this *Letter*, and two recent works [27,28], [2] the above discussion makes it clear that the conceptual foundations of the theory of general relativity and quantum mechanics are so rich that they impose concrete modifications onto each other in the interface region. Yet, a complete theory of quantum gravity shall carry "quantum" and "gravity" with new meanings - meanings that are yet to be deciphered from theory and observations in their entirety.

Acknowledgements.

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